AN ANALYSIS OF
SERVMART INVENTORY CONTROL POLICIES

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THESIS

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ABSTRACT

An investigation is made of possible inventory control policies for Navy supply system SERVMARTS (Self Service Retail Stores). Three mathematical models of the system are developed and a comparison of the solutions of these models with three current inventory control methods as applied to SERVMART demand data is presented. Based on this comparison, a stochastic, lost sales model is recommended and procedures for its implementation are discussed.

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I. INTRODUCTION

The object of this thesis is to develop an appropriate policy for inventory control in the operation of U.S. Navy SERVMARTS (Retail Self Service Stores). In keeping with the simplified supply and accounting operations for SERVMARTS, the inventory control policy should be relatively simple, understandable, and above all implementable.

Although the profit-seeking motives of private enterprise are lacking, the operation of a SERVMART store is in many ways similar to the operation of a supermarket. Items carried by a SERVMART are displayed on shelves or bins for customer selection, and all transactions with customers are conducted on a money-value basis rather than on a line-item-quantity-of-issue basis associated with usual requisitioning procedures in the Navy supply system. Line item stock records at SERVMARTS are non-existent except for a relatively few, specifically designated items; however, accountability is maintained through financial inventory records. The stores are established by Naval Supply Centers (NSC's) in close proximity to their customers in order to provide economical and expeditious supply support in items commonly used by those customers. The Naval Supply Systems Command (NAVSUP) prescribes the following criteria for SERVMART items [Ref. 1]:

(1) Unit price of less than $250.00.

(2) Average demand of at least two units per month

over a twelve month period.

(3) Though individual stock levels may be flexible, the total average on hand inventory investment value is limited to 30 days of stock.

(4) No insurance, repairable, critical, or classified items.

As can be seen by the above criteria, the items carried are of a relatively high demand and a relatively low unit cost.

The following information is provided to give some idea of the volume and range of business conducted at the SERVMARTS of a parent NSC. Approximately 18,000 line items are distributed by NSC San Diego to its twelve SERVMARTS. These stores account for about 29% of the dollar sales of Navy Stock Account (NSA) materials and about 62% of the line item issues made at NSC San Diego [Ref. 2].

To meet the NAVSUP criterion (3) above, some SERVMART managers use the simple inventory control policy of maintaining an average of 30 days stock for each item held in inventory. Obviously, such a policy leaves room for improvement, for demand rates and unit costs differ significantly among the items even though military worth or essentiality can be considered the same for all SERVMART items. In view of the characteristics of SERVMART inventories, the methods of scientific inventory control can be applied while maintaining the simplicity of a supermarket-type operation.

Section II of this thesis considers various assumptions about the SERVMART system and lists notation of parameters and variables used throughout the paper. In Section III

three models, considered applicable to the system, are developed and discussed, while the detailed derivations of these models are shown in Appendix A. The results of applying these models to data from three of the twelve SERVMARTS established by NSC San Diego are analyzed in Section IV. A summary of these results for selected items are recorded in Appendix B. A lost sale (Q,r) model is recommended for implementation in Section V and a FORTRAN program for this model is included as Appendix C. Also in Section V, the criteria for stocking items is discussed.

II. DESCRIBING THE SYSTEM

A. MAJOR ASSUMPTIONS

From the type of SERVMART operations usually encountered, the following assumptions can be made:

(1) Procurement leadtime for supply is known and is the same for all items at a SERVMART, since the source of resupply is either the parent NSC or a local business, which maintains reserve stocks to fill both the SERVMART requirements and the demands of other activities who do not deal at the SERVMART stores.

(2) The unit cost of an item is independent of the quantity ordered. Unit costs for items carried in the Navy supply system are constant regardless of the quantity ordered, and NSC contracts and purchase agreements with local businesses for items not carried in the supply system are usually for a specified unit price.

(3) The cost of placing an order for resupply is the same for all items in a SERVMART. The procedures involved in reorder are almost the same for all items, that is, visual inspection of a bin or shelf and preparation of a requisition document.

(4) The inventory carrying charge is the same for all items. Handling and storage requirements do not differ significantly among most of the items. Though some items may be more susceptible to pilferage or breakage, such additional carrying charge is difficult to evaluate and is considered to be relatively insignificant.

(5) Demands for items which occur when a SERVMART is out of stock result in lost sales. Since the customer may obtain the items directly from the parent NSC, unfilled demands are not backordered at the SERVMART. Though the exact cost of a lost sale may be difficult to derive, it may be approximated by the difference between the cost of issuing an item at a SERVMART and the cost of issuing that item at the parent NSC. Since this approximation should be nearly the same for all SERVMART items, with the possible exception of large or bulky items, it is assumed that the cost of lost sales is the same for all items.

(6) Items do not become obsolete.

(7) Essentiality of all items is the same.

(8) Procurement leadtime is so short that not more than one order is outstanding at a time for any item.

(9) Since the SERVMART manager is aware of the transactions taking place at his SERVMART on a daily basis, if only by visual inspection of the bins and shelves, the inventory system is assumed to be a continuous review system.

B. PARAMETERS AND VARIABLES

Throughout this paper the following notation will be used:

λ_i	the expected annual demand for item i
C_i	the unit cost of item i
Q_i	the order quantity for item i
r_i	the reorder point for item i, as determined from on hand stock
A	the order cost of any item
I	the inventory carrying charge of any item
TL	the procurement leadtime for any item
π	the cost of a lost sale
$\mu_i=(TL)(\lambda_i)$	the mean leadtime demand for item i
Y_i	a random variable representing demands during a leadtime for item i
N	the number of items in a SERVMART inventory
B	the budget constraint on the aggregate inventory investment equal to the value of an average of 30 days stock in each item as determined by sales during the recent past

III. DEVELOPMENT OF MODELS

A practical method of solving inventory control problems of this nature is to develop a reorder point-reorder quantity policy from a mathematical model formulated to minimize the expected value of operating costs subject to certain constraints. The constraints frequently encountered in military inventory systems are those associated with (1) a dollar investment in inventory and (2) workload (the number of resupply actions or orders to be handled in some period of time). In almost all businesses which hold inventories, there is some limit to the amount of money available for investment in inventory. The SERVMART operation is no exception as NAVSUP criterion (3) specifies. However, the second type of constraint appears to be less applicable to the SERVMART operation.

An inventory control policy which stocks an average of 30 days of stock for each item is creating an unnecessarily large number of resupply actions which could impose workload problems on the parent NSC. However, almost any sort of economic order policy constrained by a dollar value of inventory investment, whether or not constrained by the number of resupply orders permitted, will result in a significant reduction of resupply actions. Placing a specific constraint on the number of SERVMART resupply orders presupposes: (1) there is a maximum number of orders from all customers

which can be handled effectively by the parent NSC and (2) without a restriction on the number of SERVMART orders, more than a "fair share" of the NSC capabilities will be devoted to filling the SERVMART orders at the expense of other customer demands. Nevertheless, it is to be remembered that one SERVMART resupply action will probably result in many relatively cheap customer resupply actions which would have otherwise been placed on the NSC directly and thereby, have increased the parent NSC workload many-fold. For this reason, it seems inconsistent with the establishment of SERVMARTS to place a specific constraint on the number of SERVMART resupply orders, provided an economic order policy has been instituted.

A. MODEL 1

If there were no uncertainty concerning customer demands at a SERVMART, it would be a simple matter to formulate a model to minimize the variable operating costs subject to a constraint on average inventory investment. Of course, there would be no lost sales costs for if demands were known with certainty lost sales could be avoided. When developing an economic order policy on the basis of certainty of demands, the object is to minimize the costs of ordering and holding items to satisfy customer demands subject to a dollar limit on the average value of that inventory. Following the development of Hadley and Whitin [Ref. 3], the problem is stated as:

$$\text{minimize} \quad A \sum_{i=1}^{N} \frac{\lambda_i}{Q_i} + I \sum_{i=1}^{N} \frac{C_i Q_i}{2}$$

$$\text{subject to} \quad \sum_{i=1}^{N} \frac{C_i Q_i}{2} \leq B$$

Using the Lagrangian technique to solve this problem gives the optimal order quantity for item i as:

$$Q_i = \frac{2B}{\sum_{i=1}^{N} (\lambda_i C_i)^{1/2}} \left(\frac{\lambda_i}{C_i}\right)^{1/2}$$

The reorder point is simply the expected leadtime demand. A detailed derivation of this model is presented in Appendix A.

Note that neither the Lagrangian multiplier nor the inventory carrying charge appear explicitly in the equation for Q_i. In many inventory systems of this type, the inventory carrying charge is the most difficult parameter to ascertain with any degree of accuracy. However, since the formulation of this model is the same as the formulation of the Wilson EOQ model except for the inventory investment constraint, it is possible to obtain an imputed inventory carrying charge by comparing the two models. The reorder quantity formula of the Wilson EOQ model, for a particular value of the inventory carrying charge, will yield values of Q_i equal to values obtained from the reorder quantity formula of this model. The imputed inventory carrying

charge is determined by equating the two reorder quantities and solving for I as follows:

$$\frac{2B}{\sum_{i=1}^{N} (\lambda_i C_i)^{\frac{1}{2}}} \left(\frac{\lambda_i}{C_i}\right)^{\frac{1}{2}} = \left(\frac{2\,\lambda_i\,A}{I\,C_i}\right)^{\frac{1}{2}}$$

yielding

$$I^{\frac{1}{2}} = \left(\frac{A}{2}\right)^{\frac{1}{2}} \frac{\sum_{i=1}^{N} (\lambda_i C_i)^{\frac{1}{2}}}{B}$$

The numerical value of this imputed inventory carrying charge is often significantly higher than the value of I determined empirically or estimated. This suggests if this model is applicable, the inventory investment constraint may be too binding thus forcing the opportunity cost portion of the carrying charge to be much higher than ordinarily expected.

B. MODEL 2

The preceding model was based on an assumption that demands over time are deterministic, but this is seldom the case in reality. Rather, demand for item i during a time interval of length t is a random variable which is assumed to be described by a normal distribution with mean $\lambda_i t$ and variance $\sigma_i^2 t$ in the stochastic models which follow. For the lack of sufficient demand data from the SERVMARTS, some assumption about the distribution of demand is required.

Various studies have shown that the demand processes are often best described by some member of the family of compound Poisson distributions. However, for high and moderate demand items, the reorder quantity as determined by the compound Poisson distribution has been shown [Ref. 4] to be closely approximated by the reorder level which is determined by using the normal distribution having the same mean and variance. In fact, the difference in the two reorder levels rarely exceeds one. Furthermore, the use of the normal distribution for these moderate and high demand items is consistent with current Navy regulations.

An approximate, but practical and intuitively appealing, unconstrained mathematical model to determine a (Q,r) policy based on economic considerations involving lost sales has been developed by Hadley and Whitin [Ref. 3] for a single-item inventory. Under the assumptions above, the formulation may be easily modified to account for a multi-item inventory with a constraint on the value of average inventory investment. The objective is to minimize the sum of ordering costs, holding costs, and costs of lost sales subject to a limit on the dollar value of investment in average inventory. Mathematically this model is formulated:

$$\text{minimize} \quad A \sum_{i=1}^{N} \frac{\lambda_i}{Q_i} + I \sum_{i=1}^{N} C_i\left(\frac{Q_i}{2} + r_i - \mu_i\right) + \sum_{i=1}^{N} \left(IC_i + \pi\frac{\lambda_i}{Q_i}\right) W_i$$

$$\text{subject to} \quad \sum_{i=1}^{N} C_i\left(\frac{Q_i}{2} + r_i - \mu_i\right) \leq B$$

This formulation has been derived in detail in Appendix A and leads to a solution where:

$$Q_i = \left[\frac{2\lambda_i(A + \pi W_i)}{C_i(I + \theta)}\right]^{\frac{1}{2}}$$

and

$$-\frac{\partial W_i}{\partial r_i} = \int_{\frac{r_i-\mu_i}{\sigma_i}}^{\infty} \phi_i(y_i)dy_i = \frac{C_iQ_i(I + \theta)}{IC_iQ_i + \pi\lambda_i}$$

where θ is a Lagrangian multiplier and

$$W_i = (\mu_i - r_i) \cdot \int_{\frac{r_i-\mu_i}{\sigma_i}}^{\infty} \phi_i(y_i)dy_i + \sigma_i\phi_i\left(\frac{r_i-\mu_i}{\sigma_i}\right)$$

At first glance it may appear that this model presents an implementable solution where the optimum values of Q_i and r_i are obtained by an iteration process similar to that suggested by Hadley and Whitin. The Lagrangian multiplier must be adjusted simultaneously to achieve the budget constraint. In practice a solution cannot always be obtained for an inventory system composed of many items such that the cost of a lost sale is known and is the same for all items. The first partial derivative of W_i with respect to r_i is a probability having a value between zero and one; however, it is possible in attempting to meet the budget constraint that θC_iQ_i could become larger than $\pi\lambda_i$ thereby allowing $\partial W_i/\partial r_i$ to become greater than one. Consequently, for this

model, either or both of two previous assumptions are not valid: (1) the cost of a lost sale is not the same for all items, in which case it may be necessary to empirically determine a cost of a lost sale for each individual item, a formidable task that will not necessarily insure that $\pi_i \lambda_i$ be larger than $\theta C_i Q_i$ for each item, or (2) if the lost sale is indeed the same for all items, then in order to achieve the budget constraint it may be necessary to set the cost of lost sales much larger than expected from empirical determination. This last approach was used to calculate order quantities and reorder points utilizing data from three SERVMARTS at NSC San Diego. While Model 2 has the desirable feature of providing the best solutions for a lost sales formulation, it requires considerable effort to obtain values for θ and π which meet the inventory investment constraint.

Another problem area in the above formulation is the requirement that the inventory carrying charge be an input. While the inventory carrying charge may be the same for all items in a SERVMART inventory, it can be a difficult task to pin-down a value for I. To by-pass this difficulty and the difficulty of obtaining proper values of θ and π as discussed above, a third model is considered.

C. MODEL 3

As has been previously discussed, customer demands which cannot be satisfied at the SERVMARTS ultimately are placed

on the parent NSC increasing its workload. Therefore, it seems reasonable that stockouts at the SERVMARTS should be kept to a minimum in order to provide the most economical and expeditious service to customers. Clearly, the number of lost sales at a SERVMART for any item is a function of the reorder point and the number of orders per year is a function of the reorder quantity. These ideas can be used to formulate a model similar to Model 2 where the objective is to minimize the sum of the ordering costs, holding costs, and costs of lost sales subject to an inventory constraint and a risk of stockout. Although this model is developed in detail in Appendix A, the initial formulation is given here:

$$\text{minimize } A \sum_{i=1}^{N} \frac{\lambda_i}{Q_i} + I \sum_{i=1}^{N} C_i\left(\frac{Q_i}{2}+r_i-\mu_i\right) + \sum_{i=1}^{N}\left(IC_i+\pi\frac{\lambda_i}{Q_i}\right) W_i$$

$$\text{subject to (1) } \sum_{i=1}^{N} C_i\left(\frac{Q_i}{2} + r_i - \mu_i\right) \leq B$$

$$\text{(2) } \mathrm{Prob}[Y_i \geq r_i] = 1 - P, \ \forall_i$$

This formulation is the same as that of Model 2, except for the second constraint. Constraint (2) is an explicit statement about the acceptable risk (1-P) of a lost sale for an item; that is, the acceptable probability (P) that leadtime demand (Y_i) does not exceed the reorder point (r_i). Using the assumption that demands may be represented by the realizations of a normal distribution, this constraint provides a

method of determining the reorder level r_i independently of the reorder quantity, simply by specifying a value for P. Since the reorder level may be determined explicitly prior to solving for the reorder quantity, the reorder quantity Q_i can be easily solved by introducing the value for r_i into the following formula:

$$Q_i = 2\left(\frac{\lambda_i}{C_i}\right)^{\frac{1}{2}} \frac{(A + \pi W_i)^{\frac{1}{2}}\left[B + \sum_{i=1}^{N} C_i(\mu_i - r_i)\right]}{\sum_{i=1}^{N} \left[C_i \lambda_i (A + \pi W_i)\right]^{\frac{1}{2}}}$$

Note that in this model, as in Model 1, the inventory carrying charge does not appear explicitly in the formula for r_i or Q_i. The difficulty of determining a value for the inventory carrying charge has been eliminated, as has the need to iterate to obtain the optimal solution. Although this model has eliminated most of the undesirable features of the other models, a problem of determining an appropriate value for P remains. Model 2 is considered to produce favorable values for Q_i and r_i for a stochastic, lost sales model; therefore, the value of P that permits Model 3 to exhibit results comparable to those of Model 2 could be adopted as an appropriate criterion for choosing P. Applying these two models to SERVMART data provided by NSC San Diego indicates that value of P to be about 0.99. Alternatively, the choice of P could be left at the complete discretion of the SERVMART manager.

IV. RESULTS

The models discussed in the preceding section and three inventory control methods known to be in use have been applied to data from three NSC San Diego SERVMARTS to compute reorder levels, reorder quantities, and the number of orders per year for items in the inventories. Calculations were made using the total population of SERVMART C (150 items) and SERVMART L (122 items) while a random sample of 200 items was used from SERVMART J (population of 2886 items). The random sample selected represented about 7% of both the population and the average monthly sales of SERVMART J. A summary of these calculations for Models 2 and 3 has been recorded in Appendix B for selected items for the SERVMARTS. Also included in Appendix B are the results for selected items at SERVMART J as determined by Model 1 and the three existing methods. The three existing inventory control methods examined were:

(1) Method 1

An average of 30 days stock is maintained for each item held in inventory. This is a deterministic policy where the reorder level is assumed to be equal to the lead-time demand and the reorder quantity is 60 days stock. Of course for some items, adjustment to the reorder level is necessary to account for the uncertainty of demands.

(2) Method 2

The reorder level is assumed to be one-half months supply and the reorder quantity is given as 30 days stock for each item. This policy clearly meets the NAVSUP budget constraint and probably requires no adjustment of the reorder level for procurement leadtime is considerably less than one-half month.

(3) Method 3

This is a stratification model based on the high correlation between dollar sales for items in inventory and the numbers of orders per year for those items. The inventory is stratified as follows:

(a) items having at least $400 of sales in six months are reordered in quantities equal to 30 days stock,

(b) items having less than $60 of sales in six months are reordered in quantities equal to 12 months stock,

(c) all other items are reordered in quantities equal to 3 months stock.

As in Method 2 the reorder level is assumed to be one-half months supply. Obviously, there is no assurance that this method will meet the NAVSUP inventory investment constraint, and indeed for most SERVMART inventories it will not.

Based on information supplied by NSC San Diego, the parameters in these calculations were set at the following values:

order cost A = $10.00

procurement leadtime TL = 0.02 years

inventory carrying charge I = 0.15

cost of a lost sale π = \$6.00

In addition to the value of \$6.00, the cost of a lost sale in Model 3 was also set at various values over a range of reasonable estimates to conduct a check on the model's sensitivity to that parameter. In Model 2, the cost of a lost sale was forced to be a value which would allow the model to meet the inventory investment constraint. As stated before, P was set at 0.99 in Model 3 calculations summarized in Appendix B.

A comparison of the results of applying the above models and methods to SERVMART data shows:

(1) Method 3 produces an inventory control policy which results in the least number of orders per year for a SERVMART, but it fails to meet the NAVSUP inventory investment constraint. This is always true when most of the SERVMART inventory is composed of items having less than \$400 sales in six months.

(2) The economic order quantity models produce significantly fewer orders per year for a SERVMART than does either Method 2 or Method 3. This indicates that the number of issues from the parent NSC and the number of receipts at SERVMARTS may be considerably reduced if an economic order policy were instituted rather than Method 1 or Method 2. Reduction in the number of receipts at the SERVMART should permit faster stowage of items into bins or shelves and

allow the SERVMART personnel more time to closely observe inventory levels, thereby reducing the number of stockouts.

An analysis of the results of applying the three models of this study to SERVMART data reveals:

(1) Reorder points, rounded to the nearest whole number, are 15-20% lower for Model 3 than those of Model 2. Raising the value of P narrows these differences, but at the same time it lowers the values of the reorder quantities.

(2) Reorder quantities, rounded to the nearest whole number, for Model 3 are within 10% of those of Model 2.

(3) Specifying the acceptable risk of a stockout in Model 3 is in some way equivalent to specifying an additional cost of a lost sale over and above the value of π. As could be expected, the results obtained by Model 3 are insensitive to the value of π over a range of realistic values. Therefore, the difficulty of determining an appropriate value for the cost of a lost sale need not be undertaken if it is not already known.

(4) The differences in the results obtained from Models 2 and 3 are on the order of 15 to 20% for the reorder levels and 10% for the reorder quantities. These differences remained consistent over a wide range of procurement leadtimes.

V. CONCLUSIONS

A. RECOMMENDATION

Model 3 is recommended for implementation for the following reasons:

(1) It produces an inventory control policy meeting the inventory investment constraint imposed by NAVSUP which Method 3 does not do, and it results in considerably fewer numbers of resupply orders per SERVMART than Method 1 or Method 2.

(2) It attempts to account for uncertainty of demands which is not explicitly formulated into any of the methods or Model 1.

(3) Although the values calculated for the reorder levels by Model 3 differ noticeably from those of Model 2, the reorder quantities do not differ appreciably. Furthermore, if the assumption of normally distributed demands holds well in actual practice, 99% protection against stockouts should provide sufficient coverage during a leadtime.

(4) The difficult task of determining values for the inventory carrying charge and the cost of a lost sale may be avoided.

(5) Human and computer effort is minimized. Model 3 solves directly for reorder levels and reorder quantities using the inventory investment constraint as an input parameter; whereas, Model 2 requires manual and computer effort to adjust θ and π so that the inventory investment constraint may be met.

B. IMPLEMENTATION OF THE MODEL

A simple FORTRAN program for Model 3 has been enclosed as Appendix C. This program may easily be modified to accept input information of a different format and to provide additional output.

Although this program should be able to accommodate almost all SERVMART inventories, there may be inventories which are too large. If that situation occurs, a random sample of the population may be drawn to serve as the source of determining a good method of stratifying the population. Since there is a very high correlation between items sales and the number of orders per year for those items, the results of computing reorder levels and quantities for the sample using this program should suggest ideal reorder quantity break points of the type used in Method 3 that is, for some range of sales λ_i = (exp. no. orders per year)x Q_i. The reorder levels for all items of the population would still be determined by constraint (2) of the model. The random sample used in such a stratification procedure should represent the same percentage of the population, sales and budget constraint.

If Model 3, or any economic model, is implemented there may be some minor manual adjustments necessary for relatively high cost items having relatively high demands. The calculation of these models may determine reorder quantities, for those items, which are very low and which require resupply actions more frequently than a leadtime. It seems reasonable that one would desire to manually adjust these reorder quantities upward to cover at least a leadtime's demand even though such action results in an inventory which slightly exceeds the NAVSUP investment constraint.

C. STOCKING CRITERIA

Upon implementation of the recommended model, it may be necessary for economic reasons to modify the stocking criteria set forth by NAVSUP. In addition to the requirement that a SERVMART item have an average demand of at least two units per month over a twelve month period, it would be sensible to impose an additional requirement that items having a reorder quantity of one be removed from inventory. Because of the double-handling involved, a reorder quantity of one clearly makes the issue of that item to the end-user more expensive than if the end-user ordered the item directly from the parent NSC.

Although this paper has briefly addressed criteria for removing items from inventory, it does not address the problem of determining which items in a parent NSC inventory should be stocked in a system of many SERVMARTS dispersed over a wide geographical area and having a wide range of customer demands. In order to utilize the SERVMART system to the most economical extent, this problem should be investigated at a later time.

APPENDIX A: MATHEMATICAL DERIVATION OF MODELS

The formulation and derivation of the deterministic model (Model 1) follows:

$$\text{minimize} \quad K = A \sum_{i=1}^{N} \frac{\lambda_i}{Q_i} + I \sum_{i=1}^{N} \frac{C_i Q_i}{2}$$

(order costs) (holding costs)

$$\text{subject to} \quad \sum_{i=1}^{N} \frac{C_i Q_i}{2} \leq B$$

(average inventory investment)

The constraint is active since the budget constraint, based on average monthly sales, is so restrictive that use of the Wilson EOQ formula is prohibited for expected values of the input parameters. Therefore, to determine the optimal reorder quantities for this model, the Lagrangian multiplier method is used. The Lagrangian function is formulated:

$$L = A \sum_{i=1}^{N} \frac{\lambda_i}{Q_i} + \frac{I}{2} \sum_{i=1}^{N} C_i Q_i + \theta\left(\sum_{i=1}^{N} \frac{C_i Q_i}{2} - B\right)$$

The extreme point of K - that is, the optimal value of Q_i - is the solution of the equations:

$$(1\text{-}1) \qquad \frac{\partial L}{\partial Q_i} = - \frac{A \lambda_i}{(Q_i)^2} + \frac{IC_i}{2} + \frac{\theta C_i}{2} = 0$$

$$(1\text{-}2) \qquad \frac{\partial L}{\partial \theta} = \sum_{i=1}^{N} \frac{C_i Q_i}{2} - B = 0$$

Solving Equation (1-1) for Q_i yields

$$(1\text{-}3) \qquad Q_i = \left[\frac{2A\lambda_i}{C_i\ (I + \theta)}\right]^{\frac{1}{2}}$$

Substituting this value of Q_i into Equation (1-2) gives

$$(I + \theta)^{\frac{1}{2}} = \frac{1}{B} \sum_{i=1}^{N} \left(\frac{A\lambda_i C_i}{2}\right)^{\frac{1}{2}}$$

Now replacing $(I + \theta)^{\frac{1}{2}}$ in Equation (1-3) produces the desired formula for the reorder quantity

$$Q_i = \frac{2B}{\sum_{i=1}^{N} (\lambda_i C_i)^{\frac{1}{2}}} \left(\frac{\lambda_i}{C_i}\right)^{\frac{1}{2}}$$

If it is desired to determine the value of inventory carrying charge which would allow an unconstrained Wilson EOQ formula to produce an average inventory investment equal to the budget constraint, the following steps are taken to determine that value:

$$Q_i = \frac{2B}{\sum_{i=1}^{N} (\lambda_i C_i)^{\frac{1}{2}}} \left(\frac{\lambda_i}{C_i}\right)^{\frac{1}{2}} = \left(\frac{2A\lambda_i}{IC_i}\right)^{\frac{1}{2}}$$

which yields

$$I = \frac{A}{2}\left[\frac{\sum_{i=1}^{N} (\lambda_i C_i)^{\frac{1}{2}}}{B}\right]^2$$

The development of Model 2 follows:

$$\text{minimize } K = A \sum_{i=1}^{N} \frac{\lambda_i}{Q_i} + I \sum_{i=1}^{N} C_i\left(\frac{Q_i}{2}+r_i-\mu_i+W_i\right) + \pi \sum_{i=1}^{N} \frac{\lambda_i}{Q_i} W_i$$

(order costs) (holding costs) (lost sales cost)

$$\text{subject to} \quad \sum_{i=1}^{N} C_i\left(\frac{Q_i}{2} + r_i - \mu_i\right) \leq B$$

(average inventory investment)

As in the development of Model 1, the constraint is assumed to be active, and the Lagrangian technique is used to determine optimal reorder quantities and reorder points. The Lagrangian function is

$$L = A \sum_{i=1}^{N} \frac{\lambda_i}{Q_i} + I \sum_{i=1}^{N} C_i\left(\frac{Q_i}{2}+r_i-\mu_i\right) + \sum_{i=1}^{N} \left(IC_i+\pi\frac{\lambda_i}{Q_i}\right) W_i$$

$$+ \theta\left[\sum_{i=1}^{N} C_i\left(\frac{Q_i}{2}+r_i-\mu_i\right) - B\right]$$

where for the normal distribution

$$W_i = (\mu_i-r_i) \int_{\frac{r_i-\mu_i}{\sigma_i}}^{\infty} \phi_i(y_i)dy_i + \sigma_i \phi_i\left(\frac{r_i - \mu_i}{\sigma_i}\right)$$

and

$$\frac{\partial W_i}{\partial r_i} = - \int_{\frac{r_i - \mu_i}{\sigma_i}}^{\infty} \phi_i(y_i)\, dy_i$$

σ_i is the standard deviation of leadtime demand.

Again, the optimal values of Q_i and r_i are solutions of the first partial derivatives of L:

$$(2\text{-}1) \qquad \frac{\partial L}{\partial Q_i} = - \frac{A\lambda_i}{(Q_i)^2} + \frac{IC_i}{2} - \frac{\pi\lambda_i W_i}{(Q_i)^2} + \frac{\theta C_i}{2} = 0$$

$$(2\text{-}2) \qquad \frac{\partial L}{\partial r_i} = IC_i + (IC_i + \pi\frac{\lambda_i}{Q_i}) \frac{\partial W_i}{\partial r_i} + \theta C_i = 0$$

$$(2\text{-}3) \qquad \frac{\partial L}{\partial \theta} = \sum_{i=1}^{N} C_i(\frac{Q_i}{2} + r_i - \mu_i) - B = 0$$

Solving Equation (2-1) for Q_i yields

$$(2\text{-}4) \qquad Q_i = [\frac{2\lambda_i(A + \pi W_i)}{C_i(I + \theta)}]^{\frac{1}{2}}$$

and Equation (2-2) can be rewritten

$$\frac{\partial W_i}{\partial r_i} = - \frac{C_i Q_i (I + \theta)}{IC_i Q_i + \pi\lambda_i}$$

Therefore

$$\int_{\frac{r_i-\mu_i}{\sigma_i}}^{\infty} \phi_i(y_i)dy_i = \frac{C_iQ_i(I + \theta)}{IC_iQ_i + \pi\lambda_i} .$$

An iteration process is necessary to solve for reorder points and reorder quantities since the formulation leaves Q as a function of r and vice versa.

Model 3 is developed as follows:

minimize K, the same objective function as Model 2

subject to (1) $\sum_{i=1}^{N} C_i(\frac{Q_i}{2} + r_i - \mu_i) \leq B$

(2) $\text{Prob}\ [Y_i \leq r_i] = P$.

Using the Lagrangian technique, the first partial derivatives are the same as Model 2 and again

(2-4) $$Q_i = [\frac{2\lambda_i(A + \pi W_i)}{C_i(I + \theta)}]^{\frac{1}{2}}$$

Substituting this into Equation (2-3)

$$\sum_{i=1}^{N} \frac{C_i}{2} [\frac{2\lambda_i(A + \pi W_i)}{C_i(I + \theta)}]^{\frac{1}{2}} = B + \sum_{i=1}^{N} C_i(\mu_i - r_i)$$

which yields

$$(I + \theta)^{\frac{1}{2}} = \frac{\sum_{i=1}^{N} \left[\frac{(C_i\lambda_i)(A + \pi W_i)}{2}\right]^{\frac{1}{2}}}{B + \sum_{i=1}^{N} C_i(\mu_i - r_i)}$$

Substituting back into (2-4)

$$\text{(3-1)} \qquad Q_i = 2\left(\frac{\lambda_i}{C_i}\right)^{\frac{1}{2}} \frac{(A + \pi W_i)^{\frac{1}{2}} \left[B + \sum_{i=1}^{N} C_i(\mu_i - r_i)\right]}{\sum_{i=1}^{N} \left[(C_i\lambda_i)(A + \pi W_i)\right]^{\frac{1}{2}}}$$

Since r_i can be obtained directly from constraint (2), Q_i can be obtained directly from (3-1) by using the value of r_i as an input. Assuming demands are normally distribututeed, values for the reorder points are obtained from constraint (2) as follows:

$$\text{Prob } [Y_i \leq r_i] = P$$

$$\text{Prob } \left[\frac{Y_i - \mu_i}{\sigma_i} \leq \frac{r_i - \mu_i}{\sigma_i}\right] = P$$

$$\text{Prob } \left[Z \leq \frac{r_i - \mu_i}{\sigma_i}\right] = P$$

$$r_i = \mu_i + Z_P\sigma_i$$

where Z_P is that value which is exceeded by a random variable Z having the standard normal distribution with probability P.

APPENDIX B: SUMMARY OF RESULTS

The models developed in this thesis were applied to data from three SERVMARTS at NSC San Diego, and the following four pages records the results of selected items for comparative purposes. The values of the fixed parameters, for these results, were:

order cost	A = \$10.00
inventory carrying charge	I = 0.15
procurement leadtime	TL = 0.02 years
cost of a lost sale (in Model 3)	π = \$6.00 and \$0.00
acceptable risk of a stockout	$1-P$ = 0.01

The total number of orders and average inventory investment, for the total population of SERVMARTS C and L and for the sample of SERVMART J, are given for each model.

SERVMART J (SAMPLE OF 200)

				METHOD 1			METHOD 2			METHOD 3		
FIIN	UNIT PRICE	ANNUAL SALES	λ_i	Q_i	r_i	No. ORDERS PER YR.	Q_i	r_i	No. ORDERS PER YR.	Q_i	r_i	No. ORDERS PER YR.
2890019	15.76	4381.28	278	46	6	6.0	23	12	12.0	23	12	12.1
4277448	0.51	709.92	1392	232	28	6.0	116	58	12.0	348	58	4.0
2644638	6.59	303.14	46	8	1	5.8	4	2	11.5	12	2	3.8
2873740	0.20	208.80	1044	174	21	6.0	87	43	12.0	261	43	4.0
9046690	16.50	99.00	6	1	0	6.0	1	0	6.0	6	0	1.0
2778832	1.13	51.98	46	8	1	5.8	4	2	11.5	46	2	1.0
1789200	16.33	32.66	2	1	0	2.0	1	0	2.0	2	0	1.0
2981163	6.00	24.00	4	1	0	4.0	1	0	4.0	4	0	1.0
8817582	0.24	11.04	46	8	1	5.8	4	2	11.5	46	2	1.0
1768084	0.03	8.64	288	48	6	6.0	24	12	12.0	288	12	1.0
2667449	2.79	5.58	2	1	0	2.0	1	0	2.0	2	0	1.0
2492032	0.10	1.00	10	2	0	5.0	1	1	10.0	10	1	1.0

	Method 1	Method 2	Method 3
Total Number of Orders per Year	APPROX. 1120	APPROX. 2040	439.5
Average Inventory Investment	3250.00	—	5176.44

SERVMART J (SAMPLE OF 200)

FIIN	UNIT PRICE	ANNUAL SALES	λ_1	MODEL 1 Q_1	MODEL 1 r_1	MODEL 1 No. ORDERS PER YR.	MODEL 2 Q_1	MODEL 2 r_1	MODEL 2 No. ORDERS PER YR.	MODEL 3 π=6.0 Q_1	MODEL 3 π=6.0 r_1	MODEL 3 π=6.0 No. ORDERS PER YR.	MODEL 3 π=0.0 Q_1	MODEL 3 π=0.0 r_1	MODEL 3 π=0.0 No. ORDERS PER YR.
2890019	15.76	4381.28	278	14	6	19.9	8	12	34.7	9	11	30.9	9	11	30.9
4277448	0.51	709.92	1392	172	28	8.1	98	46	14.2	106	40	13.1	106	40	13.1
2644638	6.59	303.14	46	9	1	5.1	5	3	9.2	5	3	9.2	5	3	9.2
2873740	0.20	208.80	1044	238	21	4.4	135	37	7.7	147	32	7.1	146	32	7.2
9046690	16.50	99.00	6	2	0	3.0	1	1	6.0	1	1	6.0	1	1	6.0
2778832	1.13	51.98	46	21	1	2.2	12	4	3.8	13	3	3.5	13	3	3.5
1789200	16.33	32.66	2	1	0	2.0	1	0	2.0	1	0	2.0	1	0	2.0
2981163	6.00	24.00	4	3	0	1.3	2	1	2.0	2	1	2.0	2	1	2.0
8817582	0.24	11.04	46	46	1	1.0	26	4	1.8	28	3	1.6	28	3	1.6
1768084	0.03	8.64	288	323	6	0.9	182	14	1.6	199	11	1.4	199	11	1.4
2667449	2.79	5.58	2	3	0	0.7	2	0	1.0	2	0	1.0	2	0	1.0
2492032	0.10	1.00	10	33	0	0.3	18	2	0.6	20	1	0.5	20	1	0.5

	MODEL 1	MODEL 2	MODEL 3 π=6.0	MODEL 3 π=0.0
Total Number of Orders per Year	603.9	1021.6	974.8	974.9
Average Inventory Investment	3187.82	3228.15	3257.45	3257.31

SERVMART C (POPULATION)

				MODEL 2			MODEL 3 $\pi=6.0$			MODEL 3 $\pi=0.0$		
FIIN	UNIT PRICE	ANNUAL SALES	λ_i	Q_i	r_i	No. ORDERS PER YR.	Q_i	r_i	No. ORDERS PER YR.	Q_i	r_i	No. ORDERS PER YR.
0572554	32.00	5880.00	184	4	8	46.0	3	8	61.3	3	8	61.3
2044026	85.00	1870.00	22	1	1	22.0	1	2	22.0	1	2	22.0
7202244	27.00	648.00	24	2	2	12.0	1	2	24.0	1	2	24.0
9380331	0.20	486.00	2430	169	74	14.4	153	65	15.9	152	65	15.9
0519260	3.00	336.00	112	10	6	11.2	8	6	14.0	8	6	14.0
1371597	29.00	290.00	10	1	1	10.0	1	1	10.0	1	1	10.0
9145425	142.00	284.00	2	1	0	2.0	1	0	2.0	1	0	2.0
2943881	0.20	208.40	1042	111	37	9.4	100	32	10.4	100	32	10.4
5806304	0.85	120.70	142	20	8	7.1	18	7	7.9	18	7	7.9
5287586	6.50	104.00	16	3	2	5.3	2	2	8.0	2	2	8.0
6170991	0.11	34.10	310	81	15	3.8	73	12	4.2	73	12	4.2
0131282	0.05	7.00	140	81	9	1.7	73	7	1.9	73	7	1.9
1558663	0.11	0.22	2	7	1	0.3	6	1	0.3	6	1	0.3

	Model 2	Model 3 $\pi=6.0$	Model 3 $\pi=0.0$
Total Number of Orders per Year	1143.4	1462.7	1463.2
Average Inventory Investment	4244.88	4476.66	4476.22

SERVMART L (POPULATION)

FIIN	UNIT PRICE	ANNUAL SALES	λ_i	MODEL 2 Q_i	MODEL 2 r_i	MODEL 2 No. ORDERS PER YR.	MODEL 3 $\pi=6.00$ Q_i	MODEL 3 $\pi=6.00$ r_i	MODEL 3 $\pi=6.00$ No. ORDERS PER YR.	MODEL 3 $\pi=0.0$ Q_i	MODEL 3 $\pi=0.0$ r_i	MODEL 3 $\pi=0.0$ No. ORDERS PER YR.
7825907	22.90	4900.00	214	8	9	26.7	7	9	30.6	7	9	30.6
2044238	33.27	2994.30	90	4	5	22.5	4	5	22.5	4	5	22.5
1776854	22.00	924.00	42	3	3	14.0	3	3	14.0	3	3	14.0
2044014	64.95	779.40	12	1	1	12.0	1	1	12.0	1	1	12.0
0519260	3.25	669.50	206	18	10	11.4	18	9	11.4	18	9	11.4
2943747	0.17	321.64	1892	236	60	8.0	243	52	7.8	242	52	7.8
8480783	51.00	102.00	2	1	0	2.0	1	1	2.0	1	1	2.0
2666665	8.25	66.00	8	2	1	4.0	2	1	4.0	2	1	4.0
8926214	1.70	51.00	30	10	3	3.0	10	2	3.0	10	2	3.0
6338255	0.17	28.90	170	71	9	2.4	72	8	2.4	72	8	2.4
1073305	7.65	15.30	2	1	0	2.0	1	1	2.0	1	1	2.0
9002885	0.17	0.68	4	11	1	0.4	11	1	0.4	11	1	0.4

	MODEL 2	MODEL 3 $\pi=6.00$	MODEL 3 $\pi=0.0$
Total Number of Orders per Year	663.5	723.7	724.5
Average Inventory Investment	3810.58	3875.27	3874.95

APPENDIX C: A FORTRAN PROGRAM OF MODEL 3

The FORTRAN program used to determine reorder points and reorder quantities for Model 3 is given below. This program was written to accept data of the format provided by NSC San Diego, but it may be easily changed to accept a different type of input data.

Variable names used in the program are defined as follows:

ISERV(I)	the SERVMART considered
FSN(I)	the Federal Stock Number of item i
UPRICE(I)	the unit cost of item i
VALUE(I)	the annual sales of item i
DEMAND(I)	the annual demand of item i
QTY(I)	the reorder quantity of item i
R(I)	the reorder point of item i
ORDERS(I)	the number of orders per year of item i
ORDS	the total number of orders per year for all items in the inventory
VALIN	the average value of the inventory; since the reorder points and reorder quantities have been rounded to the nearest whole number, this is a check to see if the budget constraint has been met.

NDTRI is a subroutine package held in the IBM 360 source library at the U.S. Naval Postgraduate School. The input to the subroutine is P the cumulative normal probability, and the outputs are X the standard normal deviate, D the

density function value at X and IE an error code indicating whether P is between zero and one inclusive.

The FORTRAN program is:

```
      REAL * 8 FSN
      DIMENSIØN ISERV(N),FSN(N),UPRICE(N),VALUE(N),DEMAND(N),
     1QTY(N),R(N),ØRDERS(N),CCC(N)
      N=number of items in the inventory
      B=value of the budget constraint
      TL=procurement leadtime
      SC=cost of a lost sale
      A=order cost
      P=0.99
      ØRDS=0.0
      VALIN=0.0
      DENOM=0.0
      CT=0.0
      CALL NDTRI(P,X,D,IER)
      DØ 10 I=1,N
      READ(5,1000)ISERV(I),FSN(I),UPRICE(I),VALUE(I)
 1000 FØRMAT(I2,F11.0,F5.2,F7.2)
      DEMAND(I)=VALUE(I)/UPRICE(I)
      VAR=DEMAND(I)*TL
      STD=SQRT(VAR)
      R(I)=X*STD+VAR
      H=(STD*D)-(1.0-P)*(R(I)-VAR)
      CØMA=A+SC*H
      CCC(I)=SQRT(CØMA)
      CT=CT+UPRICE(I)*(VAR-R(I))
      DENØM=DENØM+SQRT(DEMAND(I)*UPRICE(I)*COMA)
   10 CØNTINUE
      DØ 20 I=1,N
      DØE=SQRT(DEMAND(I)/UPRICE(I))
      SNUM=2.0*DØE*CCC(I)*(B+CT)
      SQTY=SNUM/DENØM
      IF(SQTY.LT.0.5) GØ TØ 500
      IQTY=SQTY+0.5
      QTY(I)=IQTY
      GØ TØ 600
  500 QTY(I)=1.0
  600 ØRDERS(I)=DEMAND(I)/QTY(I)
      ØRDS=ØRDS+ØRDERS(I)
      VAR=DEMAND(I)*TL
      IRPT=R(I)+0.5
      R(I)=IRPT
      VALIN=VALIN+UPRICE(I)*(QTY(I)/2.0+R(I)-VAR)
   20 CØNTINUE
      WRITE(6,1100)
 1100 FØRMAT('1',3X,'SERVMART',8X,'FSN',8X,'UPRICE',9X,'VALUE',
     27X,'DEMANDS',6X,'QTY',8X,'NØ.ØRDERS',8X,'R')
```

```
      DØ 30 I=1,N
      WRITE(6,1200)ISERV(I),FSN(I),UPRICE(I),VALUE(I),
     3DEMAND(I),QTY(I),ØRDERS(I),R(I)
 1200 FØRMAT(6X,I2,7X,F12.0,5X,F6.2,6X,F7.2,7X,F6.1,5X,
     4F8.1,5X,F5.1,8X,F5.1)
   11 CØNTINUE
      WRITE(6,1300)ØRDS,VALIN
 1300 FØRMAT('1',3X,'TØTAL NØ. ØRDERS',F8.2,10X,'AVE.
     5VALUE ØF INVENTORY',F10.2)
      STØP
      END
      SUBRØUTINE NDTRI(P,X,D,IE)
      IE=0
      X=0.99999E+74
      D=X
      IF(P)1,4,2
    1 IE=-1
      GØ TØ 12
    2 IF(P-1.0)7,5,1
    4 X=-0.999999E+74
    5 D=0.0
      GØ TØ 12
    7 D=P
      IF(D-0.5)9,9,8)
    8 D=1.0-D
    9 T2=ALØG(1.0/(D*D))
      T=SQRT(T2)
      X=T-(2.515517+0.802853*T+0.010328*T2)/(1.0+1.432788*
     1T+0.189269*T2+0.001308*T*T2)
      IF(P-0.5)10,10,11
   10 X= -X
   11 D=0.3989423*EXP(-X*X/2.0)
   12 RETURN
      END
```

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13. ABSTRACT

An investigation is made of possible inventory control policies for Navy supply system SERVMARTS (Self Service Retail Stores). Three mathematical models of the system are developed and a comparison of the solutions of these models with three current inventory control methods as applied to SERVMART demand data is presented. Based on this comparison, a stochastic, lost sales model is recommended and procedures for its implementation are discussed.

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